Solutions to - JEE MAINS PAPER - 2 (2025, Jan 30) ①

Q1. If $\sqrt{24-10i} = \alpha + i\beta$, $\alpha, \beta \in R$ then $(\alpha^2+\beta^2)$ is equal to

(1) 29 (2) 24 (3) 22 (4) 26

Sol^n Sbs $24-10i = (\alpha^2-\beta^2)+2i\alpha\beta$

$\alpha^2-\beta^2 = 24$, $2\alpha\beta = -10$

Using $(\alpha^2+\beta^2)^2 = (\alpha^2-\beta^2)^2 + 4\alpha^2\beta^2$

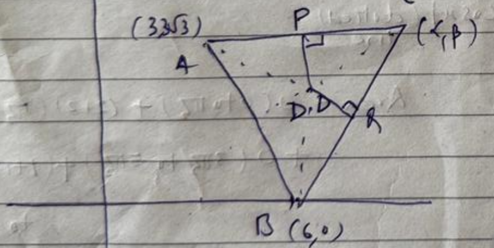
$(\alpha^2+\beta^2)^2 = (24)^2 + (-10)^2 = 576 + 100 = 676$

$\alpha^2+\beta^2 = \underline{\underline{26}}$ $\boxed{\text{option 4}}$

Q2. Let $A(3, 3\sqrt{3})$, $B(6,0)$ and $C(\alpha, \beta)$ be the vertices of an equilateral triangle. Let incentre be D, and P and Q be the feet of perpendiculars drawn from the point D on AC and BC respectively. Then the area of quadrilateral CPDQ is

(1) $3\sqrt{3}$ (2) $2\sqrt{3}$ (3) $6\sqrt{3}$ (4) $8\sqrt{3}$

Sol^n



As ΔABC is equilateral triangle and D is its incentre therefore D point will also be its centroid, orthocentre and circumcentre, P, Q will be the mid points of AC and BC respectively

side of triangle $= \sqrt{(3-6)^2+(3\sqrt{3}-0)^2} = \sqrt{9+27} = 6$

area of $\Delta = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}\times 36 = 9\sqrt{3}$

Q3 If the domain of the function, $f(x) = \dfrac{1}{\log_{10}(1-2x)} + \sqrt{2x+1}$ is $[\alpha, \beta) - \{0\}$ ②

then $20(\alpha^2+\beta^2)$ equals

(1) 13 (2) 10 (3) 12 (4) 8

Sol^n. $\log_{10}(1-2x) \neq 0 \Rightarrow 1-2x \neq 10^0 \Rightarrow -2x \neq 0$, $\boxed{x \neq 0}$

$1-2x > 0 \Rightarrow 1 > 2x$, $\Rightarrow \boxed{\frac{1}{2} > x}$

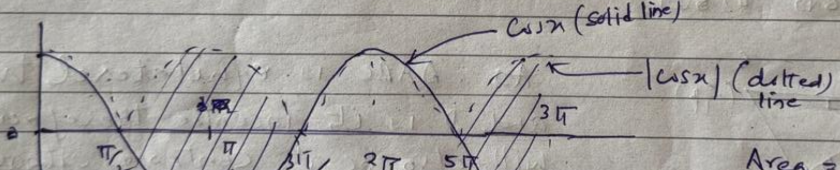
Now $\sqrt{2x+1} \Rightarrow 2x+1 \geq 0 \Rightarrow \boxed{x \geq -1/2}$ Domain is $[-\frac{1}{2}, \frac{1}{2}) - \{0\}$

$\alpha = -\frac{1}{2}$, $\beta = \frac{1}{2} \Rightarrow 20(\alpha^2+\beta^2) = 20\left(\frac{1}{4}+\frac{1}{4}\right) = \underline{10}$ $\boxed{\text{option 2}}$

Q4. The area enclosed between the curves $y = \cos x$ and $y = |\cos x|$ for $0 \leq x \leq 3\pi$ is

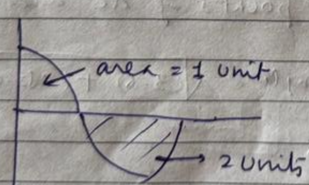
(1) 12 (2) 6 (3) 3 (4) 5

Sol^n



Area $= 0(0 \text{ to } \pi/2) + (2+2)(\pi/2 \text{ to } \frac{3\pi}{2}) + 0(3\pi/2 \text{ to } 5\pi/2) + (1+1)(\frac{5\pi}{2} \text{ to } 3\pi)$

We know



Area $= 0+4+0+2$

$= 6$

$\boxed{\text{option - 2}}$

Q5 The product of all real solutions of the equation $(3+2\sqrt{2})^{x^4-15}+(3-2\sqrt{2})^{x^2-15}=6$ is ③

(1) $4\sqrt{14}$ (2) $2\sqrt{14}$ (3) 14 (4) 18

Sol^n. We know $(3+2\sqrt{2})(3-2\sqrt{2})=1 \Rightarrow 3-2\sqrt{2}=\frac{1}{3+2\sqrt{2}}$, let $(3+2\sqrt{2})^{x^4-15}=y$

$$(3+2\sqrt{2})^{x^4-15}+\frac{1}{(3+2\sqrt{2})^{x^4-15}}=6 \Rightarrow y+\frac{1}{y}=6 \Rightarrow y^2-6y+1=0$$

$$y=\frac{6\pm\sqrt{36-4\cdot1\cdot1}}{2\cdot1}=\frac{6\pm4\sqrt{2}}{2}=3\pm2\sqrt{2}$$

Now $(3+2\sqrt{2})^{x^4-15}=(3+2\sqrt{2})^1$

$x^4-15=1 \quad x^4=16$

$x^2=\pm4 \Rightarrow \underline{x=\pm2}$

$(3+2\sqrt{2})^{x^4-15}=3-2\sqrt{2}=\frac{1}{(3+2\sqrt{2})}=(3+2\sqrt{2})^{-1}$

$x^4-15=-1$

$x^4=14 \Rightarrow x=\pm\sqrt[4]{14}$

$$\text{Product}=(2)(-2)\left((14)^{1/4}\right)\left(-(14)^{1/4}\right)=4\sqrt{14} \quad \boxed{\text{option-1}}$$

Q6 ~~Let L_1 be the line~~

Sol^n $L_1: \frac{x-1}{a}=\frac{y-2}{b}=\frac{z-(-4)}{c}$

to find $\langle a,b,c\rangle \Rightarrow \begin{vmatrix} i & j & k \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}=i(80-56)-j(-15-21)+k(24+48)$

$=24i+36j+72k=12(2i+3j+6k)$

$L_1: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

$L_2: \frac{x-7}{1}=\frac{y+6}{2}=\frac{z-0}{2}$

$$\cos\theta=\frac{(2\times1)+(3\times2)+(6\times2)}{\sqrt{4+9+36}\sqrt{1+4+4}}=\frac{2+6+12}{7\times3}=\frac{20}{21}$$

$\theta=\cos^{-1}\left(\frac{20}{21}\right)$ $\boxed{\text{option-4}}$

Q7. If the system of eq^n: $x+2y+3az=4$, $2x+3y+4z=1$, $3x+7y+2z=b$ has infinitely many sol^n then $(b-40a)$ is equal to ④

(1) 1 (2) 2 (3) 3 (4) 4

Sol^n $A=\begin{bmatrix} 1 & 2 & 3a \\ 2 & 3 & 4 \\ 3 & 7 & 2 \end{bmatrix} \Rightarrow |A|=0 \Rightarrow 1(6-28)-2(4-12)+3a(14-9)=0$

$-22+16+15a=0 \quad a=\frac{6}{15}=\frac{2}{5}$

Also $D_1=D_2=D_3=0$, $D_1=0 \Rightarrow \begin{vmatrix} 4 & 2 & 3a \\ 1 & 3 & 4 \\ b & 7 & 2 \end{vmatrix}=0$

$4(6-28)-2(2-4b)+3a(7-3b)=0$

$-88-4+8b+\frac{6}{5}(7-3b)=0 \Rightarrow -460+40b+42-18b=0 \Rightarrow 22b=418 \Rightarrow \boxed{b=19}$

Now, $b-40a=19-40\left(\frac{2}{5}\right)=19-16=3$ $\boxed{\text{option-3}}$

Q8 let $A = \{1, 2, 3 \ldots 11, 12\}$

$2x = 3y$

$R = \{(\cancel{2,3})(3,2), (6,4)(9,6)(12,8)\}$

To make it reflexive we have to add $(1,1)(2,2)(3,3) \ldots (12,12) \Rightarrow l = 12$
and to make R symmetric we have to add $(2,3)(4,6)(6,9)(8,12) \Rightarrow m = 4$

$l + m = \underline{16}$ $\boxed{\text{option-1}}$

Q9 let $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ be an ellipse of eccentricity $\frac{1}{\sqrt{3}}$. If the distance b/w the foci is 4 then length of its latus rectum is

(1) $\frac{10\sqrt{3}}{3}$ (2) $\frac{4\sqrt{3}}{3}$ (3) $\frac{8\sqrt{3}}{3}$ (4) $\frac{6\sqrt{3}}{3}$

Soln $e = \sqrt{1 - b^2/a^2} \Rightarrow \frac{1}{\sqrt{3}} = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{1}{3} = 1 - \frac{b^2}{a^2} \Rightarrow \boxed{\frac{b^2}{a^2} = \frac{2}{3}} \Rightarrow 3b^2 = 2a^2$

distance between foci $= 2ae = 4 \Rightarrow ae = 2$ also $b^2 = a^2 - a^2e^2 \Rightarrow \boxed{b^2 = a^2 - 4}$

$3(a^2 - 4) = 2a^2 \Rightarrow 3a^2 - 12 = 2a^2 \Rightarrow \boxed{a^2 = 12}$ $\boxed{b^2 = 8}$

Q10 If there are 19 and 16 points on two distinct parallel lines L_1 and L_2 respectively then the total number of triangles that can be formed using these points as vertices, is

(1) 5016 (2) 4997 (3) 4983 (4) 5049

Soln selecting 2 points on one line and one point ~~of~~ on the other line

$= {}^{19}C_2 \times {}^{16}C_1 + {}^{16}C_2 \times {}^{19}C_1 = \frac{19 \times 18}{2} \times 16 + \frac{16 \times 15}{2} \times 19 = 5016$ $\boxed{\text{option-1}}$

Q-11 let $f(x) = \sum_{r=1}^{10} \cot^{-1}(1 - (x+r) + (x+r)^2)$, $x \geq 0$ Then $\lim_{x \to \infty} (\tan(f(x)) + \sec^2(f(x)))$ is

(1) Not defined (2) 0 (3) 2 (4) 1

Soln $f(x) = \sum_{r=1}^{10} \cot^{-1}[1 - (x+r)(1 - (x+r))] = \sum_{r=1}^{10} \cot^{-1} \frac{1 - (x+r)(1-x-r)}{1}$

$= \sum_{r=1}^{10} \cot^{-1} \left\{ \frac{1 - (x+r)(1-x-r)}{(1-x-r) + (x+r)} \right\} = \sum_{r=1}^{10} \tan^{-1} \left\{ \frac{(1-x-r) + (x+r)}{1 - (x+r)(1-x-r)} \right\} = \sum_{r=1}^{10} \tan^{-1}(1-x-r) + \tan^{-1}(x+r)$

$$f(x) = \left[\left\{\tan^{-1}(-x) + \cancel{\tan^{-1}(x+1)}\right\} + \left\{\cancel{\tan^{-1}(-x-1)} + \cancel{\tan^{-1}(x+2)}\right\} + \left\{\cancel{\tan^{-1}(-2-x)} + \tan^{-1}(x+3)\right\} + \cdots + \left\{\cancel{\tan^{-1}(9-x)} + \tan^{-1}(x+10)\right\}\right]$$

$$f(x) = \tan^{-1}(-x) + \tan^{-1}(x+10) = \tan^{-1}\left(\frac{x+10-x}{1-x(x+10)}\right) = \tan^{-1}\left(\frac{10}{1-x(x+10)}\right)$$

$$\tan f(x) = \frac{10}{1-x(x+10)}, \quad \sec^2 f(x) = 1 + \tan^2 f(x) = 1 + \left(\frac{10}{1-x(x+10)}\right)^2$$

$$\lim_{x \to \infty} \frac{10}{1-x(x+10)} + 1 + \left(\frac{10}{1-x(x+10)}\right)^2 = \frac{10}{\infty} + 1 + \left(\frac{10}{\infty}\right)^2 = 1$$

$\boxed{\text{Option-4}}$

Q12 If the function $f(x)$ defined by $f(x) = \begin{cases} \dfrac{\log_e(1+5x) - \log_e(1-2x)}{x}, & x \neq 0 \\ l & x = 0 \end{cases}$

is continuous at $x = 0$ then the value of 'l' is

(1) 7 (2) 2 (3) 3 (4) 5

Soln.

$$RHL = LHL = f(0)$$

$$RHL = \lim_{x \to 0+h} \frac{\log(1+5(0+h)) - \log(1-2(0+h))}{0+h}$$

$$= \lim_{h \to 0} \frac{\log(1+5h)}{h} - \frac{\log(1-2h)}{h}$$

$$= 5 - (-2) = 7, \quad f(0) = l \Rightarrow \underline{l = 7}$$

$\boxed{\text{Option 1}}$

Q-13 In a question paper containing 10 questions A can solve 70% of question and B can solve 50% of the questions the probability that none of A and B will be able to solve the question selected at random from the paper is

(1) 3/20 (2) 2/5 (3) 3/10 (4) 1/5

Soln.

Reqd prob $= (1-P(A))(1-P(B))$ [independent events]

$= (1-0.7)(1-0.5)$

$= 0.3 \times 0.5 = 0.15 = \frac{15}{100} = \frac{3}{20}$ $\boxed{\text{Option 1}}$

Q14. Let f be a differentiable non-zero function and $3\int_0^x f(t)\,dt = 2(f(x))^3$.

Then $\int_0^1 \frac{3(f(x))^4}{\sqrt{1+(f(x))^{12}}}\,dx$ is (1) $\log_e(1+\sqrt{2})$ (2) $\log_e(3-\sqrt{2})$ (3) $\log_e(\sqrt{2}+2)$ (4) $\log_e(\sqrt{2}-1)$

Sol. Using Newton-Leibnitz Rule $3\int_0^x f(t)\,dt = 2(f(x))^3$

we get $3\,f(x)\cdot\frac{dx}{dx} - 0 = 2\cdot 3\,(f(x))^2\, f'(x) \Rightarrow 1 = 2f(x)\frac{d}{dx}f(x) \Rightarrow 1\,dx = 2f(x)\,d(f(x))$

On integration $x = (f(x))^2 \Rightarrow f(x) = \sqrt{x}$

Now $\int_0^1 \frac{3x^2\,dx}{\sqrt{1+x^6}} \Rightarrow x^3 = t \Rightarrow 3x^2dx = dt \Rightarrow \int_0^1 \frac{dt}{1+t^2} = \log(t+\sqrt{1+t^2})\Big]_0^1$

$= \log(1+\sqrt{2}) - \log 1 = \log(\sqrt{2}+1)$ $\boxed{\text{option-1}}$

Q15 If the mean of frequency distribution is 5, then its variance is

x_i	1	3	4	7	9	Total
f_i	1	a	b	3	1	10

(1) 5 (2) 5.1 (3) 5.3 (4) 5.2

Sol. $\Sigma f = 10 \Rightarrow 1+a+b+3+1 = 10 \Rightarrow \boxed{a+b=5}$ —(i)

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 5 = \frac{1+3a+4b+21+9}{10} \Rightarrow \boxed{3a+4b=19}$ —(ii)

solving (i) & (ii) we get $a=1$ $b=4$

Variance $(\sigma^2) = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$

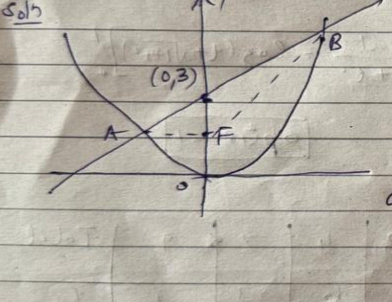
$= \frac{1+9a+16b+49\times3+81\times1}{10} - (5)^2 = \frac{1+9+64+147+81}{10} - 25 = \frac{302}{10} - 25$

$\sigma^2 = 5.2$ $\boxed{\text{option-4}}$

Q16 Let the line $y = x+3$ intersect the parabola $x^2 = 4y$ at the points A & B. If F is the focus of the parabola, then the area of ΔFAB is:

(1) 6 (2) 8 (3) 12 (4) 4

Sol.



solving $y = x+3$ & $x^2 = 4y \Rightarrow x^2 = 4(x+3)$

$x^2 - 4x - 12 = 0 \Rightarrow x = -2, 6 \Rightarrow A(-2,1),\ B(6,9)$

$F(0,1)$

ar. $\Delta ABD = \left|\frac{1}{2}\left[(0\times1-(-2)\times1)+(-18-6)+(6-0)\right]\right|$

$= \left|\frac{1}{2}[2-24+6]\right| = \underline{8}$

$\begin{array}{cc} 0 & 1 \\ -2 & 1 \\ 6 & 9 \\ 0 & 1 \end{array}$

$\boxed{\text{option-2}}$

Q17 If $\int_4^9 \frac{x-\sqrt{x}}{x(x-1)}\,dx = \alpha$, then e^α is equal to

(1) 16/9 (2) 4/9 (3) 9/4 (4) 4/3

Sol. $I = \int_4^9 \frac{\sqrt{x}(\sqrt{x}-1)}{x(\sqrt{x}-1)(\sqrt{x}+1)}\,dx \Rightarrow I = \int_4^9 \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$, let $x = t^2$, $dx = 2t\,dt$

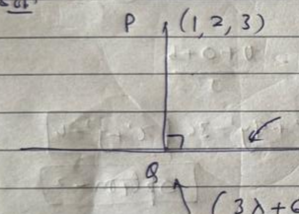
$$I = \int_2^3 \frac{2\,dt}{t(1+t)} = 2\left[\log(1+t)\right]_2^3 = 2\left[\log 4 - \log 3\right] = \log_e\left(\frac{4}{3}\right)^2 = \alpha$$

or $\frac{16}{9} = e^{\alpha}$ $\boxed{\text{option -1}}$

Q-18 The length of the perpendicular from the pt. $(1,2,3)$ on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

(1) 5 (2) 6 (3) 4 (4) 7

Sol^n



$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$ dr's of line $\langle 3, 2, -2\rangle$

$(3\lambda+6,\ 2\lambda+7,\ -2\lambda+7)$

dr's of PQ $\langle 3\lambda+5,\ 2\lambda+5,\ -2\lambda+4\rangle$

as line & PQ are $\perp^r$ $\Rightarrow 3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)=0$

$9\lambda+15+4\lambda+10+4\lambda-8=0$ $\boxed{\lambda=-1}$

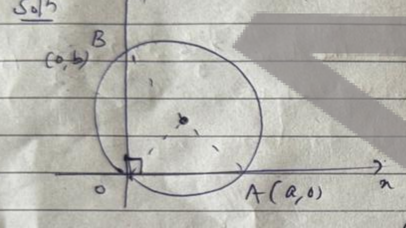
Pt. $Q \Rightarrow (3,5,9)$

$PQ = \sqrt{(3-1)^2+(5-2)^2+(9-3)^2} = \sqrt{4+9+36} = \underline{\underline{7}}$ $\boxed{\text{option-4}}$

Q19 Let a circle of radius 3 pass through the origin and meet the coordinate axes at the points A & B. Then the locus of centroid of the ΔAOB is:

(1) $x^2+y^2=1$ (2) $x^2+y^2=4$ (3) $x^2+y^2=12$ (4) $3x^2+3y^2=4$

Sol^n



Let $A(a,0)$ & $B(0,b)$

then centre of circle $\left(\frac{a}{2}, \frac{b}{2}\right)$

$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2} = 3 \Rightarrow a^2+b^2=36$ — (1)

Centroid of $\Delta AOB \Rightarrow x = \frac{0+a+0}{3}$, $y = \frac{0+0+b}{3}$

$a=3x$ & $b=3y$ sub. in (1) $\Rightarrow 9x^2+9y^2=36$, $\boxed{x^2+y^2=4}$

$\boxed{\text{option-2}}$

Q-20 Let a, b, c be the real numbers such that a, 7, c, b are in AP. If $ab=3(a+b)$ then the largest value of b is:

(1) $9+6\sqrt{2}$ (2) $9+3\sqrt{2}$ (3) $9-\sqrt{2}$ (4) $9-3\sqrt{2}$

Sol^n. As a, 7, c, b in AP $\Rightarrow a+b=7+c$ $\Rightarrow a+b = 7+\frac{7+b}{2} \Rightarrow a = \frac{21-b}{2}$

also 7, c, b is in AP $\Rightarrow c = \frac{7+b}{2}$

sub. a in $ab=3(a+b)$

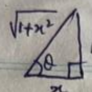
we get $b^2-18b+63=0$

Solving we get $b = 9\pm3\sqrt{2}$

Max^m value of $b = 9+3\sqrt{2}$ $\boxed{\text{option-2}}$

Q-21 If $\sum_{x=1}^{n^2} \sqrt{[x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)]^2-1} = An^4+Bn^2$ then $2(A+B)$ equals ____

Sol. Let $\cot^{-1}x=\theta \Rightarrow x=\cot\theta$,



$$\text{LHS} = \sum_{x=1}^{n^2}\sqrt{[x\cos\theta+\sin\theta]^2-1} = \sum_{x=1}^{n^2}\sqrt{\left[x\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right]^2-1} = \sum_{x=1}^{n^2}\sqrt{x^2+1-1} = \sum_{x=1}^{n^2} x$$

$$= 1+2+3\ldots+n^2$$

$$= \frac{n^2(1+n^2)}{2} = An^4+Bn^2 \Rightarrow A=\frac{1}{2},\ B=\frac{1}{2} \Rightarrow 2(A+B)=2 \quad \boxed{\text{Ans}=2}$$

Q-22 Let $A=\begin{bmatrix}1&2&3\\0&2&3\\0&0&1\end{bmatrix}$ and let P be 3×3 matrix. If $AP=\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$

and $[1\ 0\ 2]P\begin{bmatrix}1\\0\\2\end{bmatrix}=[k]_{1\times1}$ then k is ____

Sol. $|A|=2$

$$\text{adj}A=\begin{bmatrix}2&0&0\\-2&1&0\\0&-3&2\end{bmatrix}^t \Rightarrow A^{-1}=\frac{1}{2}\begin{bmatrix}2&-2&0\\0&1&-3\\0&0&2\end{bmatrix},\quad AP=\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix}$$

$$A^{-1}AP = A^{-1}\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix} \Rightarrow P=\frac{1}{2}\begin{bmatrix}2&-2&0\\0&1&-3\\0&0&2\end{bmatrix}\begin{bmatrix}1&0&1\\1&1&0\\0&1&1\end{bmatrix} \Rightarrow P=\frac{1}{2}\begin{bmatrix}0&-2&2\\1&-2&-3\\0&2&2\end{bmatrix}$$

Now $[1\ 0\ 2]\cdot\frac{1}{2}\begin{bmatrix}0&-2&2\\1&-2&-3\\0&2&2\end{bmatrix}\begin{bmatrix}1\\0\\2\end{bmatrix} \Rightarrow \frac{1}{2}[0\ 2\ 6]\begin{bmatrix}1\\0\\2\end{bmatrix} \Rightarrow \frac{1}{2}[0+0+12]=[6]$

~~option~~ $\boxed{\text{Ans}=6=k}$

Q-23 If the differential equation for the family of circles passing through the points $(19,0)$ and $(-19,0)$ is $2xy\,dx+(\alpha x^2+\beta y^2+\gamma)dy=0$ then $\alpha+\beta+\gamma$ is equal to ____

Soln. Let the eqn of circle be

$x^2+y^2+2gx+2fy+c=0$

It is passing thro $(19,0) \Rightarrow 361+0+38g+c=0$
$(-19,0) \Rightarrow 361+0-38g+c=0$ $\Rightarrow c=-361,\ g=0$

Eqn of circle is $x^2+y^2+2fy-361=0 \Rightarrow 2f=\frac{361-x^2-y^2}{y}$ —— ①

On differentiation, $2x+2y\frac{dy}{dx}+2f\frac{dy}{dx}=0 \Rightarrow 2x+2y\frac{dy}{dx}+2f\frac{dy}{dx}=0$ —— ②

sub $2f$ from ① into ②

$$2x+2y\frac{dy}{dx}+\left(\frac{361-x^2-y^2}{y}\right)\frac{dy}{dx}=0$$

$$2xy\,dx+2y^2dy+(361-x^2-y^2)dy=0$$

$$2xy\,dx+dy(361-x^2+y^2)dy)=0$$

Comparing with $2xy\,dx+(\alpha x^2+\beta y^2+\gamma)dy=0$

we get $\gamma=361,\ \beta=1,\ \alpha=-1$

$$\alpha+\beta+\gamma=-1+1+361=\underline{\underline{361}}$$

Q24 If $({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + \cdots + 10({}^{10}C_{10})^2 = m({}^{n}C_m)$ then $(m+n)$ equals ___

Sol^n. $LHS = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + \cdots + 10({}^{10}C_{10})^2$

$$= \sum_{r=1}^{10} r({}^{10}C_r)^2 = \sum_{r=1}^{10} r.{}^{10}C_r.{}^{10}C_r = \sum_{r=1}^{10} r.\frac{10}{r}.{}^{9}C_{r-1}.{}^{10}C_r = 10\sum_{r=1}^{10} {}^{10}C_r.{}^{9}C_{r-1} \quad \text{— (1)}$$

Now $\sum_{r=1}^{10} {}^{10}C_r.{}^{9}C_{r-1} = {}^{10}C_1\,{}^{9}C_0 + {}^{10}C_2\,{}^{9}C_1 + {}^{10}C_3.{}^{9}C_2 + \cdots + {}^{10}C_{10}\,{}^{9}C_9 \quad \leftarrow (2)$

Consider, $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x^1 + {}^{10}C_2 x^2 + \cdots + {}^{10}C_{10} x^{10}$

$(x+1)^9 = {}^{9}C_0 x^9 + {}^{9}C_1 x^8 + {}^{9}C_2 x^7 \cdots + {}^{9}C_9$

On multiplication

$(1+x)^{19} = x^{10}({}^{10}C_1.{}^{9}C_2 + {}^{10}C_2.{}^{9}C_1 + \cdots + {}^{10}C_{10}.{}^{9}C_9) + \cdots$ other terms

$\downarrow$

Term containg x^{10}

$${}^{19}C_{10}.\cancel{x^{10}} = \cancel{x^{10}}.({}^{10}C_1.{}^{9}C_2 + {}^{10}C_2.{}^{9}C_1 + \cdots + {}^{10}C_{10}.{}^{9}C_9)$$

$LHS = 10.{}^{19}C_{10} = m\,{}^{n}C_m \Rightarrow m = 10,\ n = 9$

$$\boxed{m+n = \underline{\underline{29}}}$$

Q25. Let $\bar{a} = 2i+j-k$, $\bar{b} = 3i+2j+4k$. Let for some $\lambda, \mu \in R$, a vector $\bar{c} = \lambda\bar{a} + \mu\bar{b}$, If $\bar{a}.\bar{c} = 16$ and $\bar{b}\times\bar{c} = -12i+22j-2k$ then $|c|^2$ equals ___

Soln $\bar{c} = \lambda\bar{a} + \mu\bar{b} \Rightarrow \bar{a}.\bar{c} = \lambda\,\bar{a}.\bar{a} + \mu\,\bar{a}.\bar{b} \Rightarrow 16 = 6\lambda + 4\mu$

$\bar{b}\times\bar{c} = \lambda\,\bar{b}\times\bar{a} + \mu\,\bar{b}\times\bar{b} \Rightarrow \bar{b}\times\bar{c} = \lambda(\bar{b}\times\bar{a}) + 0 \Rightarrow \bar{b}\times\bar{c} = \lambda\begin{vmatrix} i & j & k \\ 3 & 2 & 4 \\ 2 & 1 & -1 \end{vmatrix}$

$(-12i+22j-2k) = \lambda[i(-2-4) - j(-3-8) + k(3-4)]$

$(-12i+22j-2k) = \lambda(-6i+11j-k) \Rightarrow \boxed{\lambda = 2}$

Sub. $\lambda = 2$ in $16 = 6\lambda + 4\mu$

$16 = 12 + 4\mu \Rightarrow \boxed{\mu = 1}$

$\bar{c} = 2\bar{a} + \bar{b} = 7i + 4j + 2k \Rightarrow |c| = \sqrt{49+16+4}$

$|c| = \sqrt{69} \Rightarrow |c|^2 = \underline{\underline{69}}$